Transmission line identification using PMUs

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***Abstract*—The paper is devoted to the identification of transmission lines from synchronized measurements of current and voltage phasors provided by the phasor measurement units (PMUs). The series and shunt line parameters are recursively estimated using the extended Kalman filter (EKF). Actual line temperature and reference resistance are estimated instead of actual series resistance, which is then computed from these estimates. This approach can accommodate more information and, therefore, produce better estimates of the line resistance than the traditional approaches based on the weighted least squares method. Two case studies based on data from South Moravia and East Bohemia are presented to demonstrate the effectiveness of the proposed approach.**

I. Introduction

As the demand for electricity transmission capacity increases the knowledge of transmission line parameters involving series impedance and shunt admitance is crucial to determine the line ampacity. While construction parameters are commonly valid when the line is designed, actual values differ due to line age, ambient conditions and the load of line. It is therefore necessary to compute the actual parameters to ensure effective and safe operation of the transmission line.

Several approaches for estimating the line parameters from measured data were discussed in the past. The accuracy of the estimated values is significantly influenced by measurement errors and errors in synchronization of data acquisition. Firstly, there was an effort to use measurements from the Supervisory Control and Data Acquisition (SCADA) system. However, it was not possible to reach a satisfactory solution due to considerable synchronization inaccuracy of measured data, mainly caused by the δ–criterion used for data transfer [1]. A more promising way to acquire data utilizes synchronized current and voltage phasor PMU measurements, which utilize GPS signals to guarantee high precision of mutual synchronization of measurements from both ends of the line. A number of papers deal with the utilization of synchronized phasors in wide area measurement systems, e.g. [2] and other references mentioned therein.

Contrary to the synchronization errors, which are solved on the hardware side using PMUs, impact of measurement errors should be minimized by using an apropriate model and estimation method. The weighted least squares (WLS) method is widely used for power system state and parameter estimation, where weights are commonly set to be the inverse of the measurement error variances [3]. Each time, the line parameters are estimated based on a redundant number of current and voltage measurements taken at different time instants. The number of measurements used is limited by numerical stabilityand must be sufficiently large to yield reliable instantaneous estimates of time varying parameters [1], [4]. Therefore, new approach that uses a nonlinear filtering method for parameter estimation is proposed in this paper. Instead of directly estimating actual series resistances of the line, series resistances corresponding to some reference average temperature and actual average line temperature will be estimated.

The paper is organized such that Section II describes the proposed model of the transmission line and introduces the EKF based parameter estimation method. The proposed approach is illustrated in two case studies from South Moravia and East Bohemia in Section III. The results are summarized in Section IV.

II. Transmission line parameter estimation by EKF

Average conductor temperature T at time t can be computed using the following non-steady-state heat balance formula given in the standard IEEE Std $738^{TM} - 2006$ [5]:

$$\tau\frac{dT}{dt} = \frac{1}{l}R(T)I^2(t) - q_c - q_r, \quad (1)$$
$$R(T) = R_{20}(1 + \alpha\,(T - 20)), \quad (2)$$

where

τ	total heat capacity of conductor,
l	length of conductor,
$I(t)$	absolute value of current through the series impedance at time t,
q_c	convected heat loss rate per unit length,
q_r	radiated heat loss rate per unit length,
$R(T)$	AC resistance of conductor at temperature T,
R_{20}	AC resistance of conductor at reference temperature 20^oC,
α	temperature coefficient of resistance.

Averaged conductor temperature is understood as the average between conductor core and surface temperature.

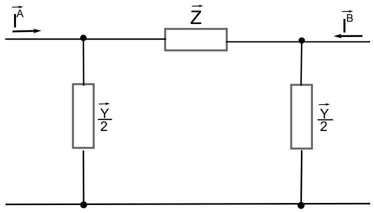

Fig. 1. Model of the transmission line.

Let us consider a three-phase transmission line, where PMUs are installed at both ends of each phase. PMUs measure, at discrete time instants k and with period Δt, voltage phasors, $\vec{U}_k = U_{Re,k} + jU_{Im,k}$, and current phasors, $\vec{I}_k = I_{Re,k} + jI_{Im,k}$. Phasors measured at one end will be marked with superscript A and at the other end with superscript B. Moreover,suppose that the ambient temperature and wind speed along the line are measured. The objective is to estimate line average temperature T_k, reference resistance $R_{20,k}$, reactance X_k, shunt conductance G_k and susceptance B_k based on measurements up to time k. Instead of independent estimating the line parameters from measurements of each of the three phases, the symmetrical component for the measured voltage and current phasors will be computed [6].

Let us define a vector $\mathbf{s}_k$ of the parameters, which represents the estimated state

$$\mathbf{s}_k = \begin{bmatrix} T_k \\ R_{20,k} \\ X_k \\ G_k \\ B_k \end{bmatrix}. \tag{3}$$

In order to use recursive estimation, equation (1) will be discretized with period Δt. Then, the discrete relation describing the development of the average temperature is as follows:

$$T_{k+1} = T_k + \frac{\Delta t}{\tau l} R_{20,k} \left(1 + \alpha \left(T_k - 20\right)\right) I_k^2 - \frac{\Delta t}{\tau} \left(q_{r,k} + q_{c,k}\right). \tag{4}$$

Since the phasor values are not measured accurately, but they are affected by measurement errors, absolute value of the current I_k through series impedance is computed as the average of the values determined from measurements at both ends of the line, which model is depicted in Fig. 1. Thus

$$I_k = \mathrm{abs}\left(\frac{1}{2}\left(\vec{I}^A - \vec{I}^B - \frac{\vec{Y}_k}{2}\left(\vec{U}^A - \vec{U}^B\right)\right)\right), \tag{5}$$

where $\vec{Y}_k = G_k + jB_k$ is the shunt admitance.

Before introducing the EKF, let us define state and measurement equations. State equations are considered as follows

$$\mathbf{s}_{k+1} = \begin{bmatrix} \mathrm{f}_1(\mathbf{s}_k) \\ \mathrm{f}_2(\mathbf{s}_k) \\ \mathrm{f}_3(\mathbf{s}_k) \\ \mathrm{f}_4(\mathbf{s}_k) \\ \mathrm{f}_5(\mathbf{s}_k) \end{bmatrix} + \begin{bmatrix} w_{1,k} \\ w_{2,k} \\ w_{3,k} \\ w_{4,k} \\ w_{5,k} \end{bmatrix} = \mathbf{f}(\mathbf{s}_k) + \mathbf{w}_k, \tag{6}$$

where

$$\begin{aligned} \mathrm{f}_1(\mathbf{s}_k) &= T_k + \frac{\Delta t}{\tau l} R_{20,k} \left(1 + \alpha \left(T_k - 20\right)\right) I_k^2 - \\ &\quad - \frac{\Delta t}{\tau} \left(q_{r,k} + q_{c,k}\right), &(7) \\ \mathrm{f}_2(\mathbf{s}_k) &= R_{20,k}, &(8) \\ \mathrm{f}_3(\mathbf{s}_k) &= X_k, &(9) \\ \mathrm{f}_4(\mathbf{s}_k) &= G_k, &(10) \\ \mathrm{f}_5(\mathbf{s}_k) &= B_k. &(11) \end{aligned}$$

Noise $\mathbf{w}_k$ makes it possible to represent uncertainty in the development of parameter values and possible time variability. As all parameters, with the exception of temperature, are assumed to be constant, $\mathbf{w}_k$ is set $\mathbf{w}_k = \mathbf{0}$ for all k.

Measurement equations are considered in the form

$$\mathbf{z}_k = \begin{bmatrix} \Delta U_{Re,k} \\ \Delta U_{Im,k} \\ \Delta I_{Re,k} \\ \Delta I_{Im,k} \end{bmatrix} = \mathbf{h}_k\left(\mathbf{s}_k\right) + \mathbf{e}_k, \tag{12}$$

$$\mathbf{z}_k = \begin{bmatrix} R_k K_{1,k} - X_k K_{2,k} \\ R_k K_{2,k} + X_k K_{1,k} \\ G_k \frac{U^A_{Re,k} + U^B_{Re,k}}{2} - B_k \frac{U^A_{Im,k} + U^B_{Im,k}}{2} \\ G_k \frac{U^A_{Im,k} + U^B_{Im,k}}{2} + B_k \frac{U^A_{Re,k} + U^B_{Re,k}}{2} \end{bmatrix} + \begin{bmatrix} e_{1,k} \\ e_{2,k} \\ e_{3,k} \\ e_{4,k} \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} K_{1,k} &= \frac{1}{2}\left[I^A_{Re,k} + I^B_{Re,k} - G_k \frac{U^A_{Re,k} - U^B_{Re,k}}{2} + \right. \\ &\quad \left. + B_k \frac{U^A_{Im,k} - U^B_{Im,k}}{2}\right], &(14) \\ K_{2,k} &= \frac{1}{2}\left[I^A_{Im,k} + I^B_{Im,k} - G_k \frac{U^A_{Im,k} - U^B_{Im,k}}{2} - \right. \\ &\quad \left. - B_k \frac{U^A_{Re,k} - U^B_{Re,k}}{2}\right], &(15) \\ R_k &= R_{20,k}\left(1 + \alpha\left(T_k - 20\right)\right), &(16) \end{aligned}$$

$\mathbf{e}_k$ denotes measurement errors of measurement instruments

$$\mathrm{E}\{\mathbf{e}_k\} = \mathbf{0},\ \mathrm{cov}(\mathbf{e}_k) = \mathbf{R} = \mathrm{diag}\left(\sigma_1^2,\ \sigma_2^2,\ \sigma_3^2,\ \sigma_4^2\right), \tag{17}$$

where variances are given by the degree of precision of the measurement instruments. As the values of $\mathbf{z}_k$ are assumed to be the differences between measurements at line ends A and B, the resulting variance is the sum of the variances of measuring instrument errors.

Uncertainty in the difference of phasors is a crucial issue in parameter estimation due to the low signal to noise ratio. Therefore it is necessary to use as much data as possible. EKF takes into account the entire measurement history up to time k over the traditional WLS approach. EKF is defined by the

TABLE I
DESIGNED AND ESTIMATED PARAMETERS OF THE LINE IN SOUTH MORAVIA

Parameter	Designed value	Estimated value
$R_{20}[\Omega]$	7.36	8.58
$X[\Omega]$	19.08	19.31
$G[\mu S]$	—	22.45
$B[\mu S]$	137.08	163.34

following relations:

$$\begin{aligned}
\mathbf{s}_{k|k} &= \mathbf{s}_{k|k-1} + \mathbf{K}_{F,k}\left(z_k - \mathbf{h}_k(\mathbf{s}_{k|k-1})\right), && (18)\\
\mathbf{K}_{F,k} &= \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left[\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}\right]^{-1}, && (19)\\
\mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{K}_{F,k}\mathbf{H}_k\mathbf{P}_{k|k-1}, && (20)\\
\mathbf{s}_{k+1|k} &= \mathbf{f}(\mathbf{s}_{k|k}), && (21)\\
\mathbf{P}_{k+1|k} &= \mathbf{F}_k\mathbf{P}_{k|k}\mathbf{F}_k^T, && (22)
\end{aligned}$$

where $\mathbf{s}_{k|k}$ is the state estimate at time k based on measurement history up to time k, $\mathbf{s}_{k+1|k}$ is the one-step prediction of the state at time $k+1$ based on measurement history up to time k,

$$\mathbf{H}_k = \frac{\partial \mathbf{h}_k}{\partial \mathbf{s}_k}\bigg|_{\mathbf{s}_k=\mathbf{s}_{k|k-1}} = \begin{bmatrix} \left(\frac{\partial \mathrm{h}_{1,k}}{\partial \mathbf{s}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{2,k}}{\partial \mathbf{s}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{3,k}}{\partial \mathbf{s}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{4,k}}{\partial \mathbf{s}_k}\right)^T \end{bmatrix},$$

$$\mathbf{F}_k = \frac{\partial \mathbf{f}}{\partial \mathbf{s}_k}\bigg|_{\mathbf{s}_k=\mathbf{s}_{k|k}} = \begin{bmatrix} \left(\frac{\partial \mathrm{f}_1}{\partial \mathbf{s}_k}\right)^T & & & & \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

III. CASE STUDIES

This section is focused on two case studies demonstrating the effectiveness of the proposed approach. Both are performed on real data from 110kV transmission lines.

A. Identification of the line in South Moravia

This subsection is devoted to parameter estimation from data measured over two days with period $\Delta t = 1s$ in South Moravia. Obtained results are summarized in Fig.2-Fig.4 and Tab.I.

B. Identification of the line in East Bohemia

This subsection is devoted to parameter estimation from data measured over six days with period $\Delta t = 1min$ in East Bohemia. Obtained results are summarized in Fig.6-Fig.7 and Tab.II.

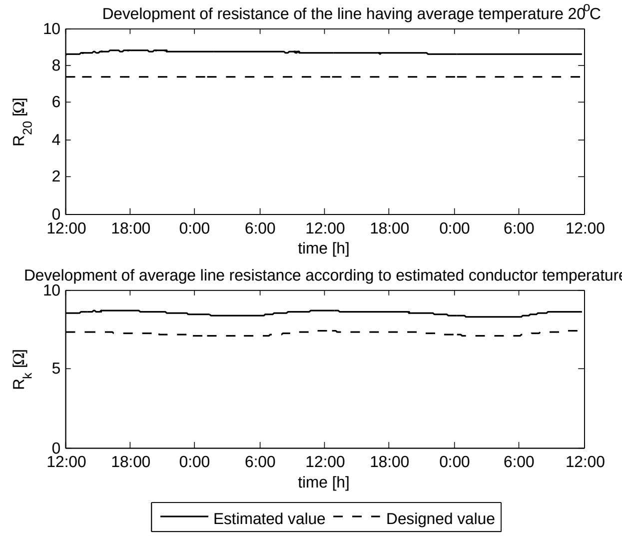

Fig. 2. Development of estimated reference resistance and computed actual resistance of the conductors according to the increasing number of measurements from the line in South Moravia.

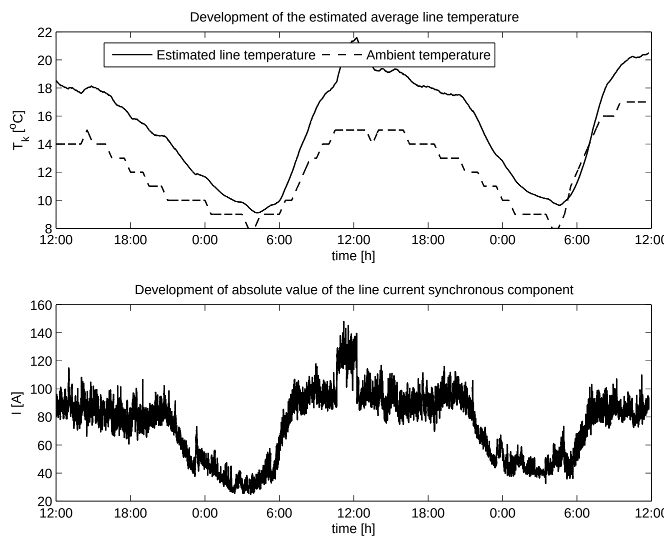

Fig. 3. Development of estimated average temperature T_k of conductor with respect to the ambient temperature and actual load I of the line in South Moravia.

TABLE II
DESIGNED AND ESTIMATED PARAMETERS OF THE LINE IN EAST BOHEMIA

Parameter	Designed value	Estimated value
$R_{20}[\Omega]$	1.56	2.02
$X[\Omega]$	7.21	7.21
$G[\mu S]$	—	2.7
$B[\mu S]$	57.78	59.52

IV. CONCLUSION

The paper was devoted to the recursive parameter estimation of transmission line using EKF. EKF is less computationally demanding, as contrary to the traditional WLS approach, there

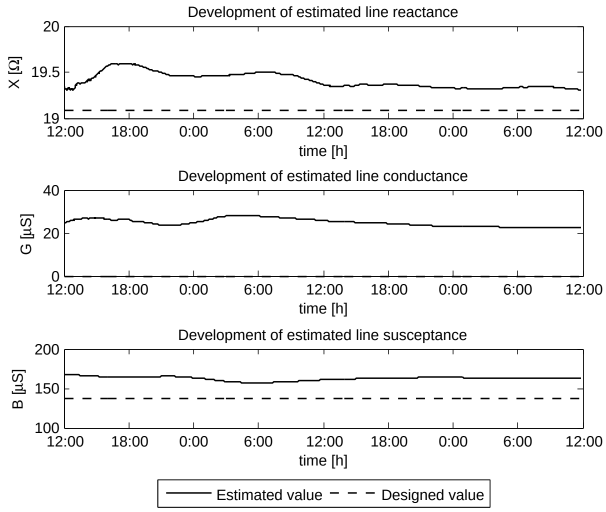

Fig. 4. Development of estimated reactance X, conductance G and susceptance B of the line in South Moravia.

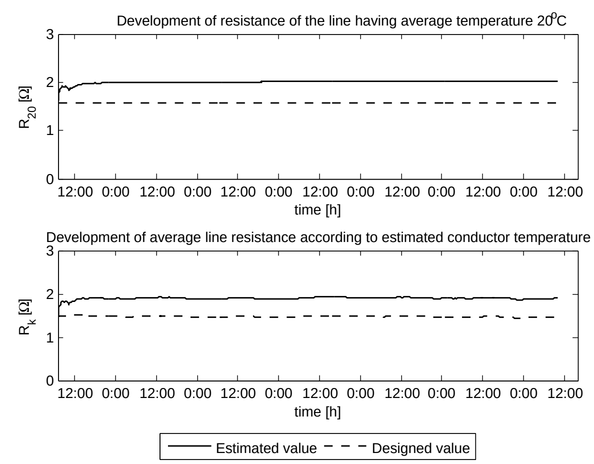

Fig. 5. Development of estimated actual resistance and computed actual resistance of conductors according to the ambient temperature and actual load of the line in East Bohemia.

is no need to compute matrix inversion at each time instant. The estimation of reference resistance and actual conductor temperature instead of actual resistance allows for the entire data history to be utilized and, thereby, improves the estimate of the actual line resistance.

Acknowledgment

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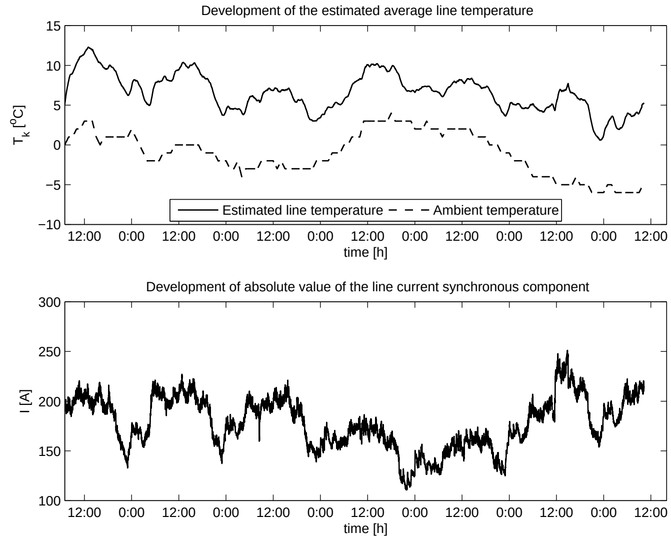

Fig. 6. Development of estimated average temperature T_k of conductor with respect to the ambient temperature and actual load I of the line in East Bohemia.

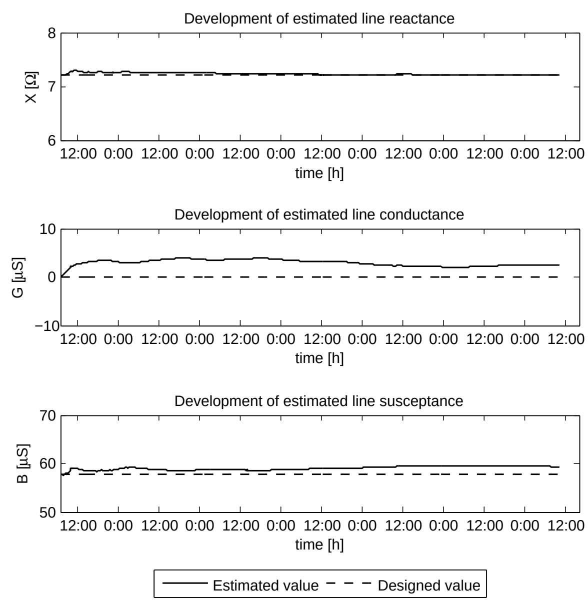

Fig. 7. Development of estimated reactance X, conductance G and susceptance B of the line in East Bohemia.

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